THE IMPACT OF THE INFLUENCE OF FOUR FOOD FACTORS AND THEIR COMBINATION IN MILK PRODUCTION IN CATTLE BY THE MATHEMATICAL MODELING

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Abstract

In this article published economic analysis of the impact of the four factors in the production of milk food and use of mathematical models in the livestock sector. The main purpose of this paper is the use of modern methods in economic analysis of use of resources in a farming complex. Livestock development in general and milk production in particular is closely linked to many factors which are the main breeding. This study analyzes the economic impact of four components ration (wet food, dry food, concentrate and mineral salts) used for milk production. The study was conducted in Lushnje district. Are analyzed and processed data feeding phases (1-up in 150 days lactation, 2-over 150 days lactation and 3- period of drying), milk production, for a period of 9 years. This study confirms that balanced nutrition is a major factor in increasing economic efficiency of farms. Another important conclusion of this study is that maximum revenue and profit maximization farm reached at the same point on the expansion path where the cost is minimal (Beattie B R, Taylor C R Beattie B R, Taylor C R, 1993). In proportion to the daily ration we have to be awarded: 59.67% wet food, dry food 27.13% and 13% koncetrat 0.20 % mineral salts of 51.42 kg food per day.

Keywords:*optimal structure, milk production, food ration. izokuant, expansion path, optimal production.*

Introduction

This study publishes the economic analyses of the impact of four nutrition factors – moisture food, dry food, concentrate and mineral salts in milk yield, as well as the use of production function in agricultural sector.

The main purpose of this study is to use contemporary methods in the economic analyses of resource usage, all this made concreate in a livestock complex.

Milk production mainly from the caws varies in different areas of Albania. Nowdays the sustainable development of agricultural farms and especially of livestock farms requires product optimization and at the same time the continuous analyses of economic and techinical impact factors.

The production function used during the study is that of Cobb-Douglas and it aims to analyse the impact of the four impact nutrition factors on caw's milk production. To be successful, dairy producers must master all aspects of dairy management. Proper dry cow nutrition and management is critical, since decisions made during this period will have a tremendous impact on milk production and health during the next lactation (Waldner D N, 1990). This study proves once more that using cattle volume nutrition system in our country's conditions makes up the primary factor for increasing the economic effectivity of farms. At the end of the study it is proved that the maximal income and profits in cattle farms are reached at the same point of expansion path where the cost is minimal.

Results and discussions:

- The production function forms for milk is determined.
- The suitability of selected models is proved.
- The optimal combination of inputs (food ration structure) to maximize gain and minimize costs is discovered.
- It is used the method of linear regression to determine the parameters of the model through the packet of econometric computerized programmes SPSS.
- It found a general formula for the optimal combination of three inputs with one output with the relevant data to a Cobb- Douglas production function.

Material and method

The livestock complex studied for this purpose was "Agrotex" in Lushnje disctrict. The data of feeding through stages was analysed and processed (1- up to 150 days of lactation, 2- over 150 days of lactation and 3- dry period), as well as data on milk yield. These data were analysed for a ten-year period.

In order to realize a more accurate dependance of the newborn calves' weight and milk quantity from inputs (food) it is procedeed according months. The average values of milk yield and average quantity of food were grouped thoughout a year (per months) according to the 3 stages of caw treatment (Edwards C H , 1994). After data processing there were built concrete function, milk yield analyses, in relation to the four production factors (moisture, dry, concentrate food and mineral salts).

The production function was requested in the following form $y = Ax_1^{\alpha} x_2^{\beta} x_3^{\gamma}$

The appeal of the Cobb-Douglas type of function rests largely with its simplicity (Debertin D L, 1986).

Linear regression method was used to determine $\log A, \alpha, \beta\, dhe\, \gamma$ through the econometric computerized programmes SPSS, from which resulted that the models are suitable. We can save predicted values, residuals, and other statistics useful for diagnostics. Each selection adds one or more new variables to your active data file (SPSS : IBM Statistics Base 19, 2010).

It came out that the models were appropriate. The presence of association does not necessarily imply causation. Statistical tests can only establish whether or not an association exists between

Variables (Mc Guigan J R, Moyer R C, Harris F H, 2008). It is confirmed the hypothesis for the importance general regression and shown that at least one of the variables provides information for prognosis of y (Myslym Osmani, 20**11)**. Based on these data, the folllowing production functions were built:

$y = 34.055788 x_1^{0.243} x_2^{0.143} x_3^{0.218} x_4^{0.025}$ where x_1, x_2, x_3, x_4, y_1 show respectively the amount of moisture, dry, concentrate food and mineral salts, average milk production,

The production function was requested in the form $y = A x_1^{\alpha} x_2^{\beta} x_3^{\gamma} x_4^{\delta}$.

We take the natural logarithms of both sides of above reconciliation

$$\ln y = \ln A + \alpha \ln x_1 + \beta \ln x_2 + \gamma \ln x_3 + \delta \ln x_4$$

It uses the linear regression method for determining of $\ln A, \alpha, \beta, \gamma$ and δ by computer econometric software package SPSS. It showed that the model is appropriate and values were found respectively: $A = 34.055788$, $\alpha = 0.243$, $\beta = 0.143$, $\gamma = 0.218$, $\delta = 0.025$

Suitability of the model

Descriptive statistics of variables and comprehensive regression results are presented in the tables below

Model Summary[b]

Model	R	R Square	Adjusted R Square	Std. Error of the Estimate	Change Statistics					Durbin-Watson
					R Square Change	F Change	df1	df2	Sig. F Change	
1	.987[a]	.974	.973	.00851	.974	981.566	4	103	.000	2.174

ANOVA[b]

Model		Sum of Squares	df	Mean Square	F	Sig.
1	Regression	.285	4	.071	981.566	.000[a]
	Residual	.007	103	.000		
	Total	.292	107			

a. Predictors: (Constant), x4, x1, x2, x3

b. Dependent Variable: y1

A generalization of the production function Coob-Douglas with four factors.

Initially, there was a generalization for the Cobb - Douglas production function $y = Ax_1^{\alpha} x_2^{\beta} x_3^{\gamma} x_4^{\delta}$ (1) giving full factor-factor and factor-product model in the general case (Themelko Henrieta, 1998).

- The izokuant equation is: $x_4 = \left(\frac{y}{A}\right)^{\frac{1}{\delta}} x_1^{-\frac{\alpha}{\delta}} x_2^{-\frac{\beta}{\delta}} x_3^{-\frac{\gamma}{\delta}}$ (2)
- The izocosts equation is: $C = p_1x_1 + p_2x_2 + p_3x_3 + p_4x_4$ (3)
- The expansion path equation is: $\frac{p_1x_1}{\alpha} = \frac{p_2x_2}{\beta} = \frac{p_3x_3}{\gamma} = \frac{p_4x_4}{\delta}$ (*)

The expansion path passes through the points of meeting "plans" izocosts (3) and "surfaces" of izoquants (2)

- The equations of "pseudo scale line" are in the following equations

$$S_1 : x_4 = \left(\frac{p_1}{A\alpha\, p_y}\right)^{\frac{1}{\delta}} x_1^{\frac{1-\alpha}{\delta}} x_2^{-\frac{\beta}{\delta}} x_3^{-\frac{\gamma}{\delta}}$$

Coefficients[a]

Model	Unstandardized Coefficients		Standardized Coefficients	t	Sig.	95.0% Confidence Interval for B		Correlations			Collinearity Statistics	
	B	Std. Error	Beta			Lower Bound	Upper Bound	Zero-order	Partial	Part	Tolerance	VIF
1(Constant)	3.528	.305		11.575	.000	2.923	4.132					
x1	.243	.051	.244	4.773	.000	.142	.344	.959	.426	.075	.095	10.497
x2	.143	.045	.312	3.146	.002	.053	.232	.977	.296	.050	.025	39.579
x3	.218	.044	.527	5.005	.000	.132	.305	.980	.442	.079	.022	44.628
x4	.025	.009	.097	2.834	.006	.007	.042	-.831	.269	.045	.214	4.676

a. Dependent Variable: y1

$$S_2 : x_4 = \left(\frac{p_2}{A\beta\, p_y}\right)^{\frac{1}{\delta}} x_1^{-\frac{\alpha}{\delta}} x_2^{\frac{1-\beta}{\delta}} x_3^{-\frac{\gamma}{\delta}} \quad (4)$$

$$S_3 : x_4 = \left(\frac{p_3}{A\gamma\, p_y} \right)^{\frac{1}{\delta}} x_1^{-\frac{\alpha}{\delta}} x_2^{-\frac{\beta}{\delta}} x_3^{\frac{1-\gamma}{\delta}}$$

$$S_4 : x_4 = \left(\frac{p_4}{A\delta\, p_y} \right)^{\frac{1}{\delta-1}} x_1^{\frac{\alpha}{1-\delta}} x_2^{\frac{\beta}{1-\delta}} x_3^{\frac{\gamma}{1-\delta}}$$

Have confirmed that four "pseudo scale lines" expected in a "point" of the expansion path.

We extracted (**)

$$C = (\alpha+\beta+\gamma+\delta)\left(\frac{p_1}{\alpha}\right)^{\frac{\alpha}{\alpha+\beta+\gamma+\delta}} \left(\frac{p_2}{\beta}\right)^{\frac{\beta}{\alpha+\beta+\gamma+\delta}} \left(\frac{p_3}{\gamma}\right)^{\frac{\gamma}{\alpha+\beta+\gamma+\delta}} \left(\frac{p_4}{\delta}\right)^{\frac{\delta}{\alpha+\beta+\gamma+\delta}} \left(\frac{y}{A}\right)^{\frac{1}{\alpha+\beta+\gamma+\delta}} \quad (**)$$

and $$y^* = \frac{A^{\frac{1}{1-(\alpha+\beta+\gamma+\delta)}} p_y^{\frac{\alpha+\beta+\gamma+\delta}{1-(\alpha+\beta+\gamma+\delta)}}}{\left(\frac{p_1}{\alpha}\right)^{\frac{\alpha}{1-(\alpha+\beta+\gamma+\delta)}} \left(\frac{p_2}{\beta}\right)^{\frac{\beta}{1-(\alpha+\beta+\gamma+\delta)}} \left(\frac{p_3}{\gamma}\right)^{\frac{\gamma}{1-(\alpha+\beta+\gamma+\delta)}} \left(\frac{p_4}{\delta}\right)^{\frac{\delta}{1-(\alpha+\beta+\gamma+\delta)}}}$$

It is proved that the profit function F has a maximum for y^* given by above equalization or C^* given by equation (**) and the values x_1^* $, x_2^*, x_3^*, x_4^*$ given by equations:

$$\begin{cases} x_1^* = \dfrac{\alpha}{\alpha+\beta+\gamma+\delta} \dfrac{C^*}{p_1} \\ x_2^* = \dfrac{\beta}{\alpha+\beta+\gamma+\delta} \dfrac{C^*}{p_2} \\ x_3^* = \dfrac{\gamma}{\alpha+\beta+\gamma+\delta} \dfrac{C^*}{p_3} \\ x_4^* = \dfrac{\delta}{\alpha+\beta+\gamma+\delta} \dfrac{C^*}{p_4} \end{cases} \qquad (5).$$

$$F_{maks} = [1-(\alpha+\beta+\gamma+\delta)\left[\left(\frac{\alpha}{p_1}\right)^{\alpha}\left(\frac{\beta}{p_2}\right)^{\beta}\left(\frac{\gamma}{p_3}\right)^{\gamma}\left(\frac{\delta}{p_4}\right)^{\delta} Ap_y\right]^{\frac{1}{1-(\alpha+\beta+\gamma+\delta)}}$$

(Sydsaeter K, Hammond P J, 1995) and (Lambert P J, 1995).

Through the latest formula can be gauged the maximum of the profit directly, using the combination of inputs with minimal cost to the level of output y^*.

It is proven that the point $(x_1^*, x_2^*, x_3^*, x_4^*)$ is in each "pseudo scale line" to (4).

- **The maximum revenues**

We form the Lagrange function of the revenues (Chiang A C, 1984):

$$L = Ax_1^{\alpha} x_2^{\beta} x_3^{\gamma} x_4^{\delta} + \lambda(C^* - p_1x_1 - p_2x_2 - p_3x_3 - p_4x_4)$$

$$\begin{cases} \dfrac{\partial L}{\partial x_1} = A\alpha\, x_1^{\alpha-1} x_2^{\beta} x_3^{\gamma} x_4^{\delta} - \lambda\, p_1 = 0 \\ \dfrac{\partial L}{\partial x_2} = A\beta\, x_1^{\alpha} x_2^{\beta-1} x_3^{\gamma} x_4^{\delta} - \lambda\, p_2 = 0 \\ \dfrac{\partial L}{\partial x_3} = A\gamma\, x_1^{\alpha} x_2^{\beta} x_3^{\gamma-1} x_4^{\delta} - \lambda\, p_3 = 0 \\ \dfrac{\partial L}{\partial x_4} = A\delta\, x_1^{\alpha} x_2^{\beta} x_3^{\gamma} x_4^{\delta-1} - \lambda\, p_4 = 0 \\ \dfrac{\partial L}{\partial \lambda} = C^* - p_1x_1 - p_2x_2 - p_3x_3 - p_4x_4 = 0 \end{cases} \qquad (***) \Rightarrow$$

(are the same solution as in equation (5))

The value of λ by the equation (***) is $\lambda = \dfrac{\alpha+\beta+\gamma+\delta}{C^*} A(x_1^*)^{\alpha}(x_2^*)^{\beta}(x_3^*)^{\gamma}(x_4^*)^{\delta}$.

We define the signs of determinant's minors and self-determinant of Hessian border.

$$|\overline{H}| = \begin{vmatrix} 0 & p_1 & p_2 & p_3 & p_4 \\ p_1 & a_1 & a_2 & b_2 & c_2 \\ p_2 & a_2 & b_1 & d_2 & e_2 \\ p_3 & b_2 & d_2 & c_1 & f_2 \\ p_4 & c_2 & e_2 & f_2 & d_1 \end{vmatrix}$$

$$|\overline{H_1}| = \begin{vmatrix} 0 & p_1 \\ p_1 & a_1 \end{vmatrix} = -p_1{}^2 < 0 \quad .$$

$$|\overline{H_2}| = \begin{vmatrix} 0 & p_1 & p_2 \\ p_1 & a_1 & a_2 \\ p_2 & a_2 & b_1 \end{vmatrix} = 2a_2 p_1 p_2 - b_1(p_1)^2 - a_1(p_2)^2 > 0$$

$$|\overline{H_3}| = \begin{vmatrix} 0 & p_1 & p_2 & p_3 \\ p_1 & a_1 & a_2 & b_2 \\ p_2 & a_2 & b_1 & d_2 \\ p_3 & b_2 & d_2 & c_1 \end{vmatrix} = -p_1 \begin{vmatrix} p_1 & p_2 & p_3 \\ a_2 & b_1 & d_2 \\ b_2 & d_2 & c_1 \end{vmatrix} + p_2 \begin{vmatrix} p_1 & p_2 & p_3 \\ a_1 & a_2 & b_2 \\ b_2 & d_2 & c_1 \end{vmatrix} - p_3 \begin{vmatrix} p_1 & p_2 & p_3 \\ a_1 & a_2 & b_2 \\ a_2 & b_1 & d_2 \end{vmatrix} < 0$$

$$\left|\overline{H}\right| = \begin{vmatrix} 0 & p_1 & p_2 & p_3 & p_4 \\ p_1 & a_1 & a_2 & b_2 & c_2 \\ p_2 & a_2 & b_1 & d_2 & e_2 \\ p_3 & b_2 & d_2 & c_1 & f_2 \\ p_4 & c_2 & e_2 & f_2 & d_1 \end{vmatrix} = \frac{(\alpha+\beta+\gamma+\delta)\alpha\,\beta\,\gamma\,\delta\,t^2 y^3}{x_1^2 x_2^2 x_3^2 x_4^2} \begin{vmatrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{vmatrix} > 0$$

We proved and the sufficient maximum conditions. It is proved that the maximum profit and maximum revenue achieved in the same point of the expansion path where the cost is minimal (Belegu M, Sallaku E, 2008).

So the production function is:

$$y = 34.055788 x_1^{0.243} x_2^{0.143} x_3^{0.218} x_4^{0.025}$$

The prices are: $p_1 = 6.8$, $p_2 = 8.8$, $p_3 = 28$, $p_4 = 210$, $p_y = 47$

- The izokuant equation is: $x_4 = \left(\dfrac{y}{34.055788}\right)^{\frac{1}{0.025}} x_1^{-\frac{0.243}{0.025}}\, x_2^{-\frac{0.143}{0.025}}\, x_3^{-\frac{0.218}{0.025}}$
- The izocosts equation is:

$$C = 6.8 \cdot x_1 + 8.8 \cdot x_2 + 28 \cdot x_3 + 210 \cdot x_4$$

- The expansion path equation is:
$$\frac{6.8x_1}{0.243} = \frac{8.8x_2}{0.143} = \frac{28x_3}{0.218} = \frac{210x_4}{0.025}$$
- The equations of "pseudo scale line" are in the following equations

$$S_1 : x_4 = \left(\frac{6.8}{34.055788 \cdot 0.243 \cdot 47}\right)^{\frac{1}{0.025}} x_1^{\frac{0.757}{0.025}} x_2^{-\frac{0.143}{0.025}} x_3^{-\frac{0.218}{0.025}}$$

$$S_2 : x_4 = \left(\frac{8.8}{34.055788 \cdot 0.143 \cdot 47}\right)^{\frac{1}{0.025}} x_1^{-\frac{0.243}{0.025}} x_2^{\frac{0.857}{0.025}} x_3^{-\frac{0.218}{0.025}}$$

$$S_3 : x_4 = \left(\frac{28}{34.055788 \cdot 0.218 \cdot 47}\right)^{\frac{1}{0.025}} x_1^{-\frac{0.243}{0.025}} x_2^{-\frac{0.143}{0.025}} x_3^{\frac{0.782}{0.025}}$$

$$S_4 : x_4 = \left(\frac{210}{34.055788 \cdot 0.025 \cdot 47}\right)^{\frac{1}{-0.975}} x_1^{\frac{0.243}{0.975}} x_2^{\frac{0.143}{0.975}} x_3^{\frac{0.218}{0.975}}$$

- In the general case is proved that the maximum of the profit is achieved if:

$$y^* = \frac{34.055788^{\frac{1}{0.371}} 40^{\frac{0.629}{0.371}}}{\left(\frac{6.8}{0.243}\right)^{\frac{0.243}{0.371}}\left(\frac{8.8}{0.143}\right)^{\frac{0.143}{0.371}}\left(\frac{28}{0.218}\right)^{\frac{0.218}{0.371}}\left(\frac{210}{0.025}\right)^{\frac{0.025}{0.371}}} = 6667.54036\, kg$$

- The minimal cost is:

$$C = 0.629\left(\frac{6.8}{0.243}\right)^{\frac{0.243}{0.629}}\left(\frac{8.8}{0.143}\right)^{\frac{0.143}{0.629}}\left(\frac{28}{0.218}\right)^{\frac{0.218}{0.629}}\left(\frac{210}{0.025}\right)^{\frac{0.025}{0.629}}\left(\frac{6667.540436}{34.055788}\right)^{\frac{1}{0.629}} \quad \text{or}$$

C=197112.4979 lekë.

So is the maximum profit per year for a cow is
$F = 47 \cdot 6667.540436 - 197112.4979 = 116261.9026$ ALL.
The cost for a kg of milk will be 29,563 = 29.6 ALL.
The profit for 1 kg milk is 17,437 ALL.

- The maximal profit is:

$$F = 0.371\left[\left(\frac{0.243}{6.8}\right)^{0.243}\left(\frac{0.143}{8.8}\right)^{0.143}\left(\frac{0.218}{28}\right)^{0.218}\left(\frac{0.025}{210}\right)^{0.025} \cdot 34.055788 \cdot 47\right]^{\frac{1}{0.371}} = 116261.9025$$

In the general case it was proved that the values of inputs that have the maximum of the profit are given by the following equations:

$$\begin{cases} x_1^* = \frac{0.243}{0.629}\frac{197112.4979}{6.8} \\ x_2^* = \frac{0.143}{0.629}\frac{197112.4979}{8.8} \\ x_3^* = \frac{0.218}{0.629}\frac{197112.4979}{28} \\ x_4^* = \frac{0.025}{0.629}\frac{197112.4979}{210} \end{cases} \quad \text{ose} \quad \begin{cases} x_1^* = 11198.52637\, kg \\ x_2^* = 5092.334007\, kg \\ x_3^* = 2439.843547\, kg \\ x_4^* = 37.30647625\, kg \end{cases}$$

so the daily ration will be: $\begin{cases} x_1^* = 30.68089416\, kg \\ x_2^* = 13.95160002\, kg \\ x_3^* = 6.684502868\, kg \\ x_4^* = 102.2095240\, gr \end{cases}$

Well will be awarded 30.68 kg of wet food, 13.95 kg dry food, 6.68 kg concentrate and 102.21 g mineral salts in a day that a cow produce 6667.54 kg of milk a year or 18:27 kg of milk per day. In proportion to the daily ration we will have to be given: 59.67 % wet food, dry food 27.13 % dry food, 13% concentrate and 0.20 % mineral salts of 51.42 kg food per day.

Conclusions

The following conclusions are attained from the sudy:

- During the process of decision-making it is becoming always more evident that it is necessary to make detailed scientific researches. Thus, the realization of livestock production necessitates the analyses of inputs in production.
- Applying Cobb-Douglas production functions gives the opportunity to realize economic analyses of farms for milk caws breeding.
- The study proved that for average production levels the most optimal structure would be: 59.67 % wet food, dry food 27.13 % dry food, 13% concentrate and 0.20 % mineral salts of 51.42 kg food per day..
- If the theoretical arguments concerning the relative effectiveness of different economic systems are subject to empirical testing, it is necessary to do some current estimates of effectiveness indicators (Luptácik M, 2008). In the general case, it is showed that maximum income is obtained for the same input amount where the maximum profit is reached.
- In conclusion, based on our country's conditions, volume system nutrition is prefered.

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